Title: Profitable Soda Stand

Brief Overview:

This unit uses a soda stand simulation to encourage problem solving and to explore the real-life meaning of intercepts, domain, range, and linear equations. Students will determine appropriate selling prices for sodas based on costs of supplies using data acquired from a local store and experimentation in the classroom. Students will use tables, graphs, and linear equations to make decisions and answer questions.

Links to NCTM 2000 Standards:

- **Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation**
  These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

- **Patterns, Functions, and Algebra**
  Students will develop and use tables to create functional relationships in graphical and abstract form. Students will use the functional relationship to apply abstract analysis to real situations.

- **Geometry and Spatial Sense**
  Students will determine appropriate scale for representing their functional relationships.

Links to Virginia High School Mathematics Core Learning Units:

- **A.2**
  The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables. Students will choose an appropriate computational technique, such as mental mathematics, calculator, or paper and pencil.

- **A.5**
  The student will analyze a given set of data for the existence of a pattern, represent the pattern algebraically and graphically, if possible, and determine if the relation is a function.

- **A.6**
  The student will select, justify, and apply an appropriate technique to graph a linear function in two variables. Techniques will include slope-intercept, x- and y-intercepts, graphing by transformation, and the use of the calculator.

- **A.15**
  The student will determine the domain and range of a relation given a graph or a set of ordered pairs and will identify the relations that are functions.

- **A.16**
  The student will, given a rule, find the values of a function for elements in its domain and locate the zeros of the function both algebraically and with a graphing calculator. The value of f(x) will be related to the ordinate on the graph.

- **AII.8**
  The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.

- **AII.9**
  The student will find the domain, range, zeros and inverse of a function, the value of a function for a given element in its domain, and the composition of multiple functions. Functions will include those that have domains and ranges that are limited and/or discontinuous. The graphing calculator will be used as a tool to assist in investigation of functions, including exponential and logarithmic.

Grade/Level:

Grades 8-12: Algebra I, Algebra II

Duration/Length:

Three 50-minute periods or two 90-minute blocks, with a pre-lab assignment.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Writing an equation of a line
- Identifying domain and range
- Using function notation
- Graphing a line
- Finding x- and y-intercepts and slope from a graph and/or equation

Objectives:

Students will:

- use problem solving to make decisions.
- use tables and graphs to extract meaningful information about a function.

- use real-world applications to give real meaning to line terminology.

Materials/Resources/Printed Materials:

- 2-liter bottles filled with water (one per group)
- Ice (with paper towels)
- Glasses of a variety of known sizes
- Liquid measuring glasses/beakers (one per group)
- Colored pencils (at least three different colors per group)

Development/Procedures:

Note: Each day is based on a 50-minute class period.

Pre-lab:
Using the survey, have the class decide what a "good" choice for drink selections may be. Have them fill this information into Table 1 on Homework Sheet 1. Put the students in groups of 3 or 4. Students need to decide who will go to which store to collect data. Recommendation: Assign this over a weekend.

Day 1:
Set up appropriate number of "filling stations" with a 2-liter bottle of water, bowl of ice (approximately 4 cups), liquid measuring cup/beaker, and paper towels. Assign and give each group a glass size to be used for their simulation. Have students fill in Part I of their plan on Group Worksheet 1 before they begin work on Activity Sheet 1.

Day 2:
Have students fill out Assignment Sheet 2. They will need all their worksheets and colored pencils to complete this activity. Discuss answers as a class, if time. Assign Homework Sheet 2.

Day 3:
Finish any discussions then give the assessment. Also included is a technology resource describing TI-83 calculator steps for graphing lines, finding x-intercepts using the Root function and finding functional values. The assessment should take approximately 20 minutes.

Assessment:

- Teacher will informally assess students by observing students as they work on activity sheets.
- A formal assessment is included.
- Scoring Rubric: For each numbered question in the assessment points will be assigned depending on the quality of the answers. 2 points will be given for correct or well-supported answers. 1 point will be given for partially correct or partially supported answers. 0 points will be given for incorrect unsupported answers.

Extension/Follow Up:

- Compare different groups' outcomes and determine who had the most profit.
- Examine groups' profit equations and determine if different groups ever had the same profit using systems of equations.
- Extend the simulation to cover multiple days introducing variable cost equations.

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Survey

What are your favorite two flavors/brands of sodas?

Would you buy a generic brand of your favorite flavor of soda? If so, which ones?

Homework Sheet 1 – How Much Is That Soda in the Window?

Name:_____

Your Lacrosse team is raising funds for a team trip to the amusement park. You decide to offer a soda stand during the game to raise the money. Your job as the "mathematicians" on the team is to find the maximum profit that can be reasonably obtained from selling your soda. You will need to determine the glass size you will use, the amount of ice in each glass, the soda brands that will be available as well as the cost per soda.

Research production costs.

Table 1 - Cost per 2 liter bottle/bag

Product	Giant	Safeway	Food Lion	Superfresh
Coke				
Pepsi				
Diet Coke				
Sprite				
Root Beer				
Orange				
Ice (weight _____)				

Table 2 – Cost per bag of glasses

Glass Size	Giant		Safeway		Food Lion		Superfresh	
	Price	#Glasses	Price	#Glasses	Price	#Glasses	Price	#Glasses
9 oz								
12 oz								
16 oz								
18 oz								

Group Worksheet 1 – What Will We Buy?

Names of Persons in your Group:_____

Group Name:_____

Part I:
Starting Plan: Assume that you are willing to pay $30 out of your pocket to get supplies. Estimate how much of each item (soda, ice, and glasses) you think you will need to purchase?

Soda Flavor/Brand:_____ Number of bottles:_____ Cost:_____

Soda Flavor/Brand:_____ Number of bottles:_____ Cost:_____

Soda Flavor/Brand:_____ Number of bottles:_____ Cost:_____

Size of Bag of Ice:_____ Number of bags:_____ Cost:_____

*Glass Size:_____ Number of glasses:_____ Cost:_____

Total Cost:_____

Go to Activity Sheet 1 – To Ice or Not to Ice

Part II:
Revised Plan: Assume that you are willing to pay $30 out of your pocket to get supplies. Based on the "To Ice or Not to Ice" Activity 1 how would you change your purchase decisions?

Soda Flavor/Brand:_____ Number of bottles:_____ Cost:_____

Soda Flavor/Brand:_____ Number of bottles:_____ Cost:_____

Soda Flavor/Brand:_____ Number of bottles:_____ Cost:_____

Size of Bag of Ice:_____ Number of bags:_____ Cost:_____

*Glass Size:_____ Number of glasses:_____ Cost:_____

Total Cost:_____

Explain why you decided to make the changes that you did. _____

_____

_____

*a "glass" represents the container in which the soda will be served

Activity Sheet 1 – To Ice or Not to Ice

Your group's glass size:_____ Group Name:_____

Determining how many glasses of soda that can be obtained from one 2-liter bottle of soda.
Note: There are three cups of ice in a one-pound bag of ice.

Amount of Glass Filled with Ice	Amount of Ice per Glass	Glasses of Ice per Bag	Amount of Soda per Glass	Glasses of Soda per Bottle
No ice				
¼ of glass				
½ of glass				
¾ of glass				

1. Fill in the Revised Plan on Group Worksheet 1 – Part II (previous page)

2. With the amount of supplies that you have chosen to purchase, how many glasses of soda can you make?

3. Determine your production cost for each soda. In other words, how much are you spending on one glass of soda?

4. What factors will affect the price that a consumer will pay for a glass of soda?

5. Choose three reasonable selling prices for your glasses of soda.

Activity Sheet 2 – Show Us the Money

Group Name:_____

Determining Profit Equations.

Number of Glasses of Soda that can be made:_____

Total Cost for Supplies:_____

Production Cost per glass:_____

Selling Price per glass: 1. _____ 2. _____ 3. _____

1. Using one of the three selling prices complete the following table.

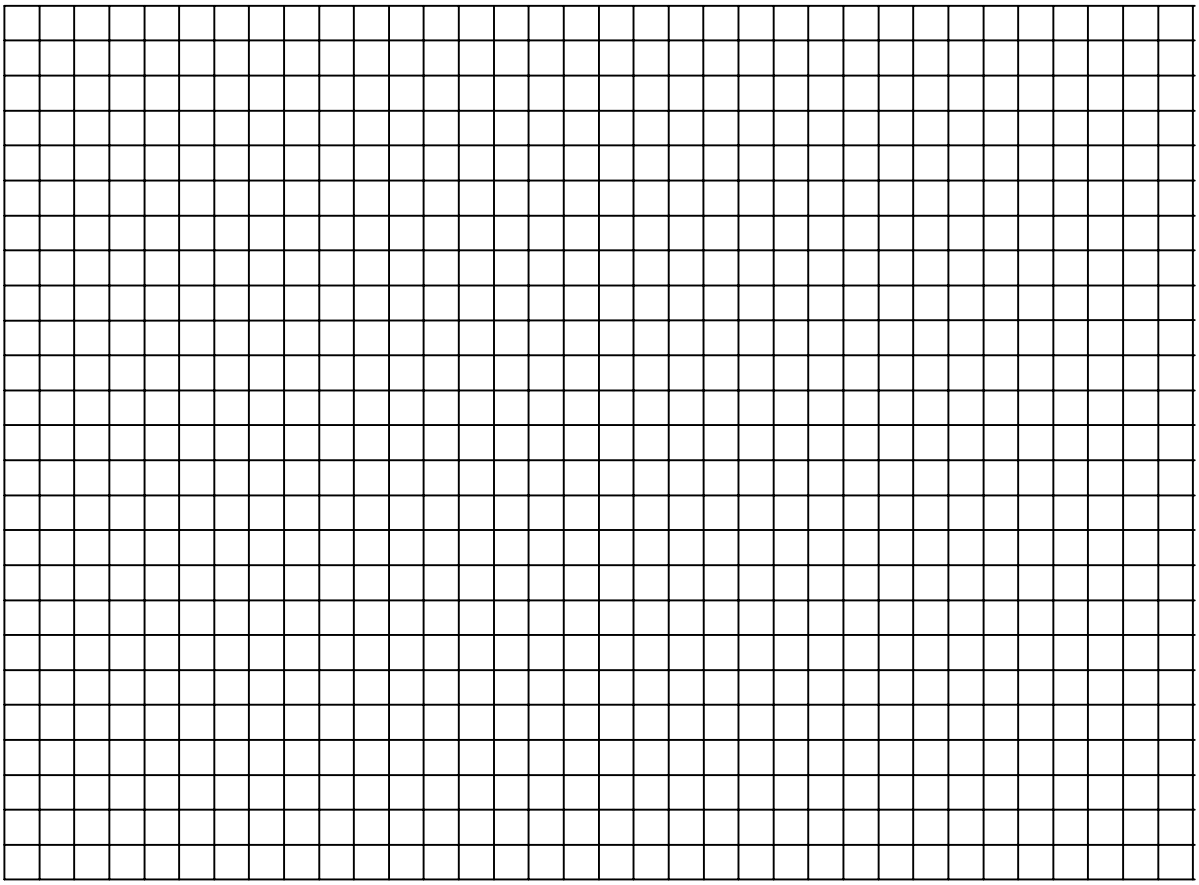
# of Glasses Sold	Income ($)	Total Costs ($)	Profit or Loss ($)
-1			
0			
1			
2			
3			
4			
5			
10			
15			
20			

2. What type of function (linear, quadratic, etc) models the soda stand's profit?

3. For the price you used in the above table, what is the profit equation as a function of the number of glasses sold?

4. What are the profit equations for the other two selling prices?

Using the information from the previous page, graph the three profit equations on the same grid. Use different colors to represent the different profits. Choose a scale that will use as much of the graph paper as possible (keep in mind the number of glasses you can sell and the profit that will then be made).



Each of the following questions will have 3 separate answers. Write each answer in the same color as its related line.

5. What are the y-intercepts for each of your three profit equations?

6. What are the x-intercepts for each of your three profit equations?

7. Do the y-intercepts have anything in common? If so, why?

8. Do the x-intercepts have anything in common? If so, why?

9. What do the y-intercepts on this graph represent?

10. What do negative y-values represent to the soda stand situation?

11. What do the x-intercepts on this graph represent?

12. What do negative x-values represent to the soda stand situation?

13. Where on the graph would you find the maximum profit for your soda stand? What value is limiting your possible profit? What is that maximum profit?

14. What is the domain for the profit equation that was created? What is the domain for the equation using the soda stand's limitations that were discussed above?

15. What is the range for the profit equation that was created? What is the range for the equation using the soda stand's limitations that were discussed above?

16. Which price gives you the most profit? Which price seems the most reasonable to you? Decide which price you are going to use to sell the glasses of soda.

Homework Sheet 2 – Do I Get to Go???

Name:_____ Date:_____

Mid-year you hear about a trip that you want to take over the summer so you decide to sell something to your classmates in an effort to start raising money (of course you get permission from the Principal first!). You discover that most people have either lost or used up their pens, pencils and notebooks already. You decide with the new semester starting soon that you would sell packs containing a new pen, pencil and spiral notebook. You can get pens for $4.80 for a pack of 24 pens and pencils for $ 2.50 for a pack of 50 pencils. Spiral notebooks come in packs of 6 for $2.00.

1. After thinking about your classmates, you decide that you will only be able to sell at most 50 packs of 1 pen, 1 pencil and 1 notebook.

    a. How many packages of pens, pencils, and notebooks should you buy?

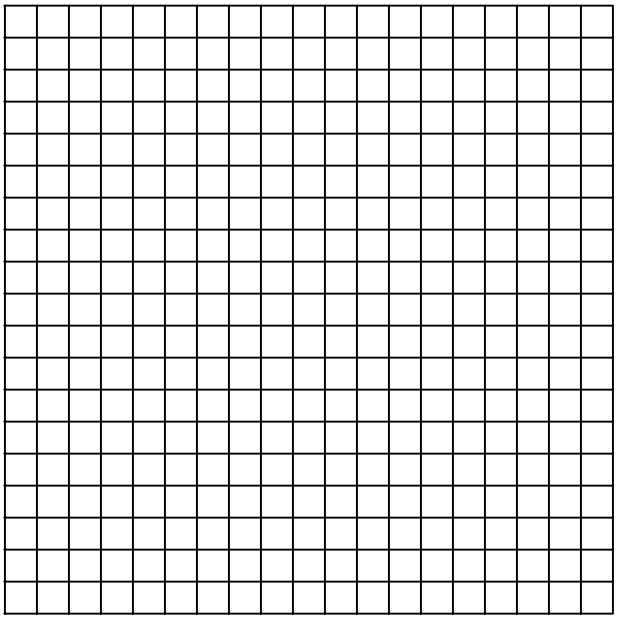
    b. What would be your total cost for supplies?

    c. What is your cost per pack?

2. You decide to sell the packages for $1.75 each. What is your profit per pack?

3. Find a profit equation with the above conditions.

4. Graph your profit equation.



5. What are the values and real-life significance of each of the following:

   a. x-intercept?

   b. y-intercept?

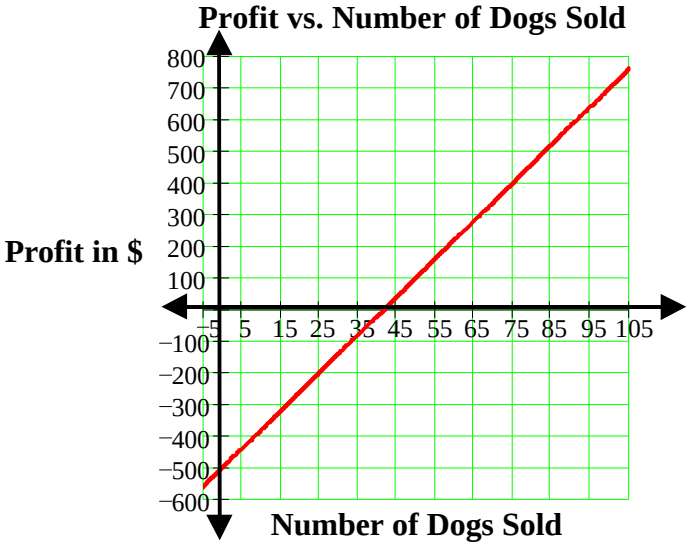
   c. domain?

   d. range?

Dogs and Candy Quiz

Name:_____

Disney finished the film *101 Dalmatians* and they need to sell the dogs to good homes. Using the graph of the profit equation shown below, answer the following questions. Show all work!



1. What is the equation of the line shown?

2. What is the selling price for each dog?

3. What was the total initial cost of the dogs?

4. What is the production cost per dog?

5. At what point did Disney break even in its costs?

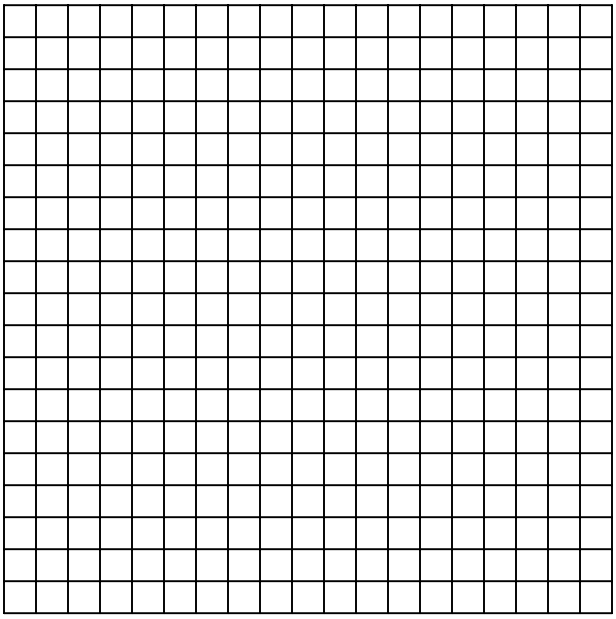
6. What is the maximum profit possible?

The Club is doing a fundraiser and decides to sell candy bars. They can get the candy bars for $0.25 each so they decide to buy 100 bars to sell. The selling price for the candy bars is $0.50 per bar. Use this information to answer the following.

7. What is the total initial cost for the candy bars?

8. What is the profit equation?

9. Graph the profit equation.

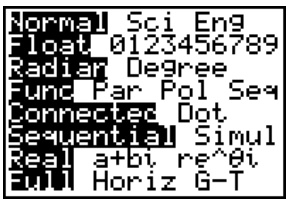


10. What is the domain of this situation?

11. What is the range of this situation?

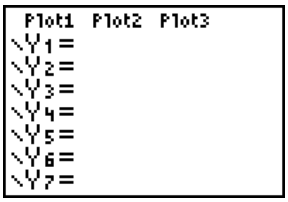
How to Graph on a TI-83 Calculator

1. Press [MODE] and ensure that everything on the left is highlighted.

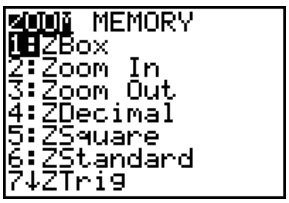


2. Press [2nd] [FORMAT] and ensure that everything on the left is highlighted.

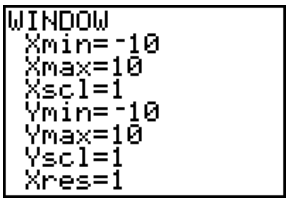
3. Press [Y=]. Type the first equation in $\backslash Y_1 =$



4. Before graphing you must set up a view screen. Start with a Standard Viewing Screen. This is found under [ZOOM] option **6: ZStandard**.

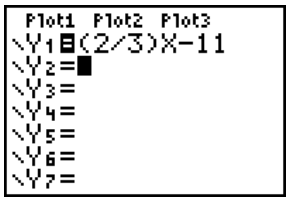


5. This viewing window may not show all of the critical points on the graph. If it does not show what you want to see, customize the window using [WINDOW]. Fill in the necessary information. (Looking in [2nd] [TABLE] may help give ideas for these values.) Xmin should be the smallest x value that you would like to see. Xmax should be the biggest x value that you would like to see. (Note: Xmin must be smaller than Xmax. If it is not set up correctly an ERR: WINDOW RANGE will show on the screen. You will need to **1:QUIT** and go back to [WINDOW] and reenter your values.) Xscl is the interval between each tick mark on the axis. Ymin, Ymax and Yscl correspond with appropriate y-values. These do not have to be the same as your x-value settings.

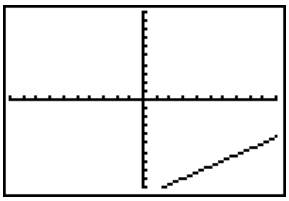


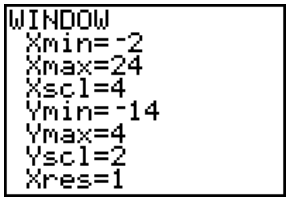
6. Press [GRAPH] to view your function. If unsatisfied with the settings, repeat step 4.

Example for graphing:

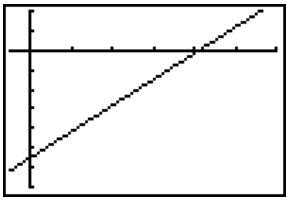


With Standard Window →



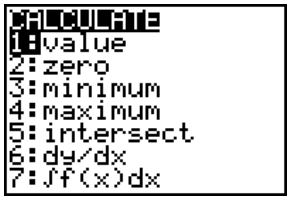


With New Window →

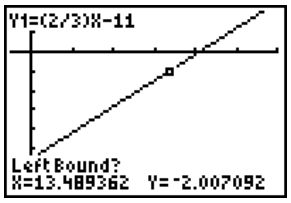


Finding x-intercepts Using a TI-83 Calculator

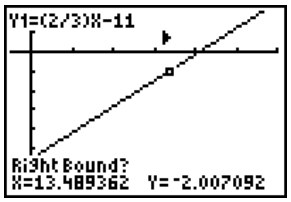
1. Press [2nd] [CALC] and select option **2: zero**. Zero makes the equation equal zero or makes y = 0. In x-intercepts, recall that y = 0.



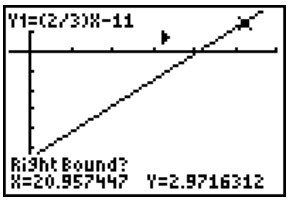
2. The calculator will request a left bound. To do this, put your cursor where you would guess the x-intercept would be and then press the left arrow key three times.



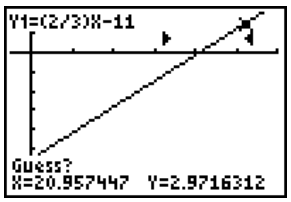
3. Press [ENTER]. A small arrow will appear on your viewing screen pointing to the right at the same x location as where you pressed enter.



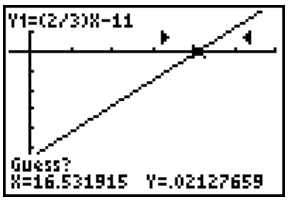
4. The calculator will request a right bound. To do this, put your cursor where you would guess the x-intercept would be and then press the right arrow key three times.



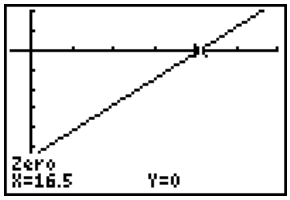
5. Press [ENTER]. A small arrow will appear on your viewing screen pointing to the left at the same x location as where you pressed enter.



6. The calculator will request a guess. To do this, put your cursor where you would guess the x-intercept would be.

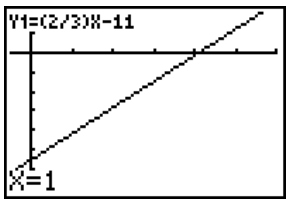


7. Press [ENTER] and the answer will be displayed at the bottom of your screen.

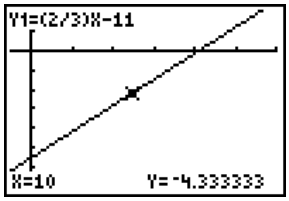


Finding a Value Using a TI-83 Calculator

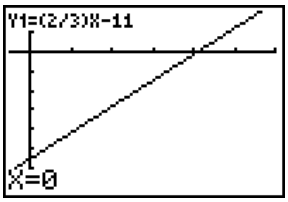
1. To find the functional value of a known x-value, press [TRACE] . Then begin typing the x-value you want.

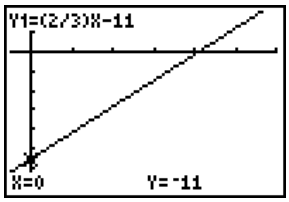


2. After inputting the x-value, press [ENTER] . The calculator will move the cursor to that point and display the x-value and corresponding y-value.

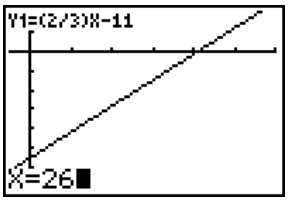


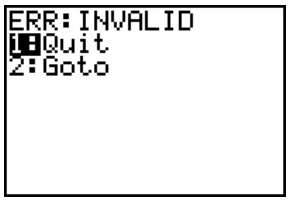
3. To find the y-intercept of a function, recall that it will have an x-value of 0.





4. If you try to find a value outside of the domain of the window, it will cause an error like the one shown. If this occurs, choose option **2: Goto** and then use a different x-value or change your viewing window settings. There is no problem if you find a y-value outside of the range of the window.





Teacher Notes

***The distinction between glasses (the item being sold) and measuring cups (a unit of measure) should be maintained to limit student confusion.*

Homework Sheet 1 – How Much is that Soda in the Window?
Assign this to students several days in advance of the experiment.

***For this activity you may find that providing the prices and sizes for the glasses will be more reasonable than having the students do the research. This can be provided in class on day one.*

Group Worksheet 1 – What Will We Buy?
Day One

Students come together to discuss and share information from Homework sheet 1
Part I:
***This part of the plan should be "best guesses". Remind students that this is only a guess. They will revise their answers later in the lab. Also remind students that they only have $30 to spend so they can<u>not</u> go over and remind them that they may use less than $30 if they wish.*

Part II:
***Possible reasons: increase glasses because had more soda than glasses, increase soda for the amount of glasses and ice, the amount of ice that will be put in each glass, etc.*

Activity Sheet 1 – To Ice or Not to Ice
Day One

***Suggestions:*
*1. A one-pound bag of ice will yield approximately 3 cups of ice. Have students measure how many cups a ¼-full, ½-full, ¾-full glass of ice uses. For example, a 9-oz glass ¼ full of ice might use ½-measuring cup of ice. The students should measure all three ice options **<u>before</u>** they pour any water into the glass.*

2. Remind students that they will only get a few trials for determining the amount of liquid in each glass with ice. They should measure the water in the measuring cup before pouring it into the glass so that they may determine the amount of liquid in the glass. Reinforce the idea that the total amount of liquid and ice in the glasses should be a consistent level. They may find marking the glass beneficial.

Determining how many glasses of soda that can be obtained from one 2-liter bottle of soda.

1. Using the above information and the prices of the soda and ice, choose the way your concession stand is going to fill the glass with soda and ice.
   ***This is an educated guess type decision.*

2. **Fill in the Revised Plan on Group Worksheet 1.** ***Remind students to go back and do this.*

3. With the amount of supplies that you have chosen to purchase, how many glasses of soda can you make?
   ***Encourage students to consider what supplies they will run out of first and how many glasses it will make. Just because the bought 200 glasses does not necessarily mean that they can sell or even fill 200 glasses of soda. There may be supplies left over. All answers are going to depend on glass size, and ice amount. There is no right or wrong answer to this question. Student's answers should be supported with appropriate computations.*

4. Determine your production cost for each soda. In other words, how much are you spending on one glass of soda?
   ***(total supply costs)/(# of possible sodas)*

5. What factors will affect the price that a consumer will pay for a glass of soda?
   ***Possible answers: amount of soda in glass, size of glass, vending machine prices, etc.*

6. Choose three appropriate selling prices for your glasses of soda.
   ***Encourage students to consider production costs, factors discussed in number 11, reasonableness, want to make a profit*

Activity Sheet 2 – Show Us the Money
Day Two

Determining Profit Equations.

1. Using one of the three selling prices complete the following table.

# of Glasses Sold	Income ($)	Cost ($)	Profit ($)
-1	***This situation does not make sense!!*		
0	*$0*	*Initial production costs*	*Income – Cost*
1	*Selling price * 1*	*“ ”*	*“ ”*
2	*Selling price * 2*	*“ ”*	*“ ”*
n	*Selling price * n*	*“ ”*	*“ ”*

2. What type of function (linear, quadratic, etc) models the soda stand’s profit?
   ***linear*

3. For the price you used in the above table, what is the profit equation as a function of the number of glasses sold?
   ***Note: Tell students if you want them to write equations in function notation or y = mx+b form or both.*
   *y = selling price * x – initial production cost where y = $ of profit and x = # of glasses sold*
   *f(x) = selling price * x – initial production cost where f(x) = $ of profit and x =# of glasses sold*

4. What are the profit equations for the other two selling prices?
   ***Slope will change, all other values will stay the same.*

Using the information from numbers 2 and 3, graph the three profit equations on the same grid. Use different colors to represent the different profits. Choose a scale that will use as much of the graph paper as possible.
***For the questions encourage students to continue using the colored pencils to help distinguish answers. For example if they drew their first equation in blue for then all the questions below they would write the answers (x-intercepts etc.) in blue also.*

5. What are the y-intercepts for each of your three profit equations?
6. What are the x-intercepts for each of your three profit equations?
   ***Depends on student decisions. Should approximate values from graph.*

7. Do the y-intercepts have anything in common? If so, why?
   ***They are all the same. Starting costs are the same.*

8. Do the x-intercepts have anything in common? If so, why?
   ***They get smaller as the price charged gets bigger. More you charge the faster you pay off the initial production cost. This may require a lot of discussion to ensure understanding.*

9. What do the y-intercepts on this graph represent?
   ***Initial production costs.*

10. What do negative y-values represent to the soda stand situation?
    ***Negative profit - Lost money. Production costs are more than income.*

11. What do the x-intercepts on this graph represent?
    ***Break even. Profit = cost. If the break-even point is a decimal, then discuss – does it make sense to sell a part of a glass of soda to a person? The profit may never equal exactly zero.*

12. What do negative x-values represent to the soda stand situation?
    ***Negative glasses of soda sold. Stolen glasses?*

13. Where on the graph would you find the maximum profit for your soda stand? What value is limiting your possible profit? What is that maximum profit?
    ***The x-value or number of glasses sold will limit profit since there is only supplies to make a certain amount of glasses. The maximum profit = the selling price * number of glasses possible – initial production costs.*

14. What is the domain for the profit equation that was created? What is the domain for the equation using the soda stand's limitations that were discussed above?
    ***Encourage the students to realize that the equation's domain is infinite; however, the situation dictates only positive integer values up to the amount of glasses that they can make with the amount of supplies they have.*

15. What is the range for the profit equation that was created? What is the range for the equation using the soda stands limitations that were discussed above?
    *** Encourage the students to realize that the equation's range is infinite; however, the situation dictates values from the negative production costs up to the maximum profit.*

16. Which price gives you the most profit? Which price seems the most reasonable to you? Decide which price you are going to use to sell the glasses of soda.
    ***This is an opinion question, educated guessing, based on the profits each of their experimental prices obtained and the reasonableness of the soda price.*

***Included is a technology resource that could be used to extend this activity. It will give the students an opportunity to get more accurate x-intercepts and maximum values, more appropriate viewing windows, and discover relationships between the increased selling price (slope) and profit.*

Homework Sheet 2 – Do I Get to Go???

Mid-year you hear about a trip that you want to take over the summer so you decide to sell something to your classmates in an effort to start raising money (of course you got permission from the Principal first!). You discover that most people have either lost or used up their pens, pencils and notebooks already. You decide with the new semester that you would sell people a new pen, pencil and spiral notebook. You can get pens for $4.80 for a pack 24 pens and pencils for $ 2.50 for a pack of 50. Spiral notebooks come in packs of 6 for $2.00.

1. After thinking about your classmates, you decide that you will only be able to sell at most 50 sets of 1 notebook, 1 pencil and 1 pen.
   a. How many packages of pens, pencils, and notebooks should you buy?
      ***2 packs of pens, 1 pack of pencils, 8 packs of notebooks*
      $9.60 + $2.50 + $16.00 = $28.10

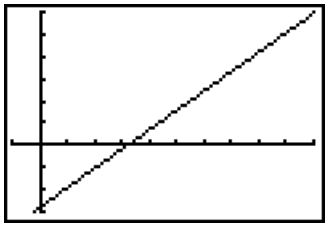
   b. What would your cost for this be?
      ***$28.10*

   c. What is the cost per set?
      $** \frac{\$28.10}{48\ packs} \approx \$0.59 / \ pack$

2. You decide to sell the packages for $1.75 each. What is your profit per set?
   ***$1.16*

3. Find a profit equation with the above conditions.
   ***P(n) = 1.75n – 28.10*

4. Graph the profit equation.



   [-5, 50: 5] [-30, 60: 10]

5. What are the values and significance of each of the following:
   a. x-intercept?
      ***(16.1, 0), break-even. Losing money 16 packs, making money at 17 packs*

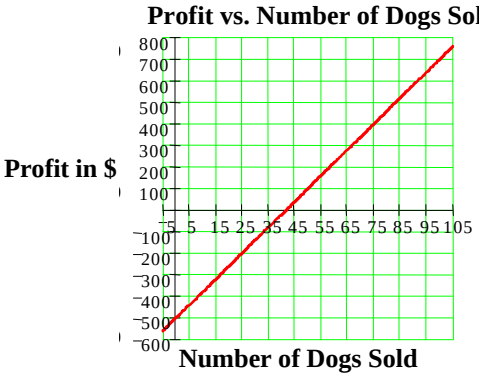
   b. y-intercept?
      ***(0, -28.10), initial production costs*

   c. domain?
      **\{ x / 0 £ x £ 48 }, minimum number of sets sold is 0 and maximum number of sets are sold is 48*

   d. range?
      **\{y / -28.10 £ y £ 55.90}, when zero sets are sold in the hole production costs, when all sets are sold make maximum profit*

Dogs and Candy Quiz

Disney finished the film *101 Dalmatians* and they need to sell the dogs to good homes. Using the graph of the profit equation shown below, answer the following questions. Show all work!



1. What is the equation of the line shown?
   ***f(x) = 12x - 500*

2. What is the selling price for each dog?
   ***$12.00*

3. What was the total initial cost of the dogs?
   ***$500.00*

4. What is the production cost per dog?
   ** $\frac{\$500}{101 dogs} \approx \$4.95 / dog$

5. At what point did Disney break even in his costs?
   ***41.7 dogs*

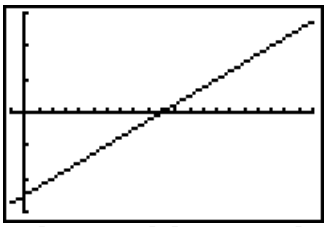
6. What is the maximum profit possible?
   ***12(101) – 500 = $712.00*

The Club is doing a fundraiser and decides to sell candy bars. They can get the candy bars for $0.25 each so they decide to buy 100 bars to sell. The selling price for the candy bars is $0.50 per bar. Use this information to answer the following.

7. What is the total initial cost for the candy bars?
   ***100 bars * $.25 = $25.00*

8. What is the profit equation?
   ***P(c) = .50c - 25*

9. Graph the profit equation.



[-5, 105: 5] [-30, 30: 10]

10. What is the domain of this situation?
    ***{x / 0 £ x £ 100}*

11. What is the range of this situation?
    ***{y / -25 £ y £ 25}*